

# Determining the Gravitational Interaction of Proposed Tertiary Star System WDS 11333+5748

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## Abstract

We studied the gravitational interactions of WDS 11333+5748 with the goal of determining if the four stars in the system are gravitationally bound as in a wide binary system. We took images of the system to perform astrometric measurements and analysis. Using the measurements, we calculated radial distance, proper motion, the Harshaw Statistic, and produced a proper motion diagram. We found that the C star is four orders of magnitude further away than the Aa, Ab, and B stars. The remaining stars are radially closed enough to be a wide tertiary system. Based on the results of our data collection and analysis, we suggest that Aa, Ab, and B are a tertiary star system while C is an optical binary

## 1. Introduction

Our research goal is to analyze proper motion, separation ( $\rho$ ), and position angle ( $\Theta$ ) to determine if WDS 11333+5748 is a binary, tertiary, or optical double/triple star system. An optical star system does not have gravitational influence with the star(s) that appear to be close yet appear nearby due to the two-dimensional nature of stellar imaging. A binary star system, on the other hand, is a gravitationally bound system that has a common center of mass and orbits that can be measured over time to determine mass, composition, and other stellar properties. This makes finding binary star systems an important part of stellar astronomy.

The target of our research, WDS 11333+5748, was chosen due to the lack of published research on the nature of the star system. It is currently cataloged as a four-star system that was originally classified as a three-star system with the AB pair discovered in 1891 and the AC pair discovered in 1898 by A. Kruger. It was most recently viewed in 2017 by the European Space Agency's (ESA) GAIA mission. The number of observations for each star pair vary, with the AB, AC, and Aa,Ab pairs being observed 14, 8, and 4 times, respectively. There has been a total of 7 publications that reference the system and measurements, but no published research that determines the gravitational nature of the stars in the system. This made it a prime candidate for study, especially given the AB and AC pairs have a differential magnitude ( $\Delta$ -mag) of 1.05 and 2.51, respectively.

Originally, this star system was cataloged as a tertiary star system, including an AB and AC pair. However, in 1989, a fourth star was found orbiting very near the main star in the system using more advanced telescopes and computing tools not available when the system was first cataloged in 1891. The companion star is so close to the main star that it is indistinguishable with standard CCD imaging, as seen in Figure 1. The historical data up to 1989 refers to the discoverer codes of the system as KR 39 AB and KR 39 AC. However, in 1989 the A-star was broken apart and labeled as Aa,Ab, with Aa being the most massive star in the system. Currently, the system's discoverer codes include the original KR 39 AB and KR 39 AC with the addition of MLR 551 Aa,Ab. The system will hereafter be referred to by the discoverer code MLR 551 for ease of reading.

For the purposes of this research, the Aa star will be primarily referenced as the A-star, unless explicitly stated. Due to the limitations of the methods used to accurately resolve the differential magnitudes ( $\Delta$ -mag), separation ( $\rho$ ), and other measurements required in analysis, Aa will be the star considered in all measurements and analysis. During the discussion, the possible effects of the Ab star on

the Aa star and how those effects may have influenced the results will be discussed. Possibilities for future research on the Aa,Ab star system will also be further discussed.

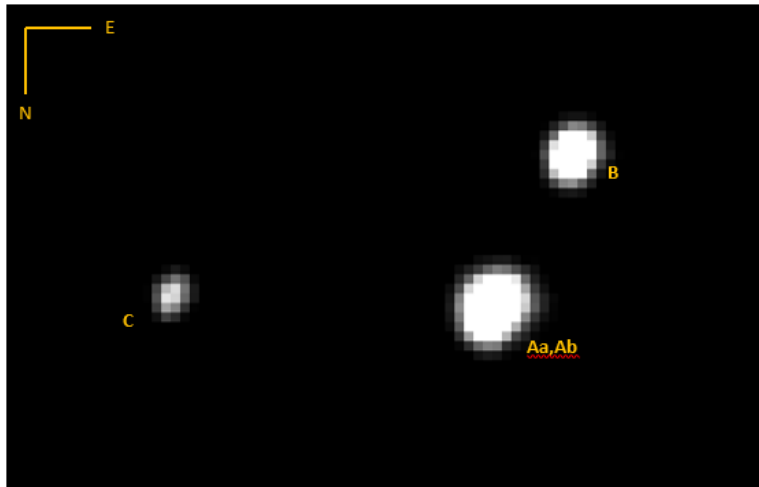


Figure 1: Image of WDS 11333+5748, labeled with the current naming convention. Notice that the Aa and Ab stars cannot be distinguished with standard imaging

## 2. Equipment and Methods

### Equipment

All images used for data analysis were collected by the two 0.4-meter telescopes at Haleakala Observatory in Maui, Hawaii. These are part of the Los Cumbres Observatory's (LCO) worldwide telescope network. The instruments on these telescopes are SBIG STL6303 CCD cameras. The Haleakala Observatory telescopes are located 10,000 feet above sea level, which is critical in reducing atmospheric aberration in photometric data (LCO 2023).

Image processing was provided through the OSS (Our Solar Sibling) Pipeline (Fitzgerald 2018). This process of image calibration removes the noise of cosmic rays, hot pixels, background radiation, etc. and flattens the images. Each image has also been processed through the World Coordinate System (WCS). These image processing techniques allow the image data to be analyzed to determine the binary nature of the star system.

### Observations

Test images were collected on 2023.263 and 2023.277 using infrared (ip) and red (rp) filters and various exposure times to determine the best calibration for collecting data on the three visible stars in the system. Table 1 shows the dates, filters, exposure times, and number of images taken during this calibration period. Cloud cover on April 6, 2023, caused the test images on this date to be dimmer than other test images, so these images were not used during the calibration process. After evaluating the remaining images and narrowing down the results, it was determined that the best time and filter combination for collecting data was 8 seconds with a red filter.

Data collection began on 2023.310, using a red filter and an exposure time of 8 seconds. During this data collection run, 14 images were taken. The second data run was taken on 2023.315. During this run, 14 images were taken with the same red filter and an exposure time. After the images were taken by the observatory, the images underwent final calibration through the OSS Pipeline and returned to us for analysis.

### *Analysis Procedures*

Data analysis began with importing the images into AstroImageJ (AIJ) software for plate solving (Collins 2017). Once the images were properly oriented in the sky with correct right ascension (RA) and declination (DEC), measurements could be made within the software. Aperture size was set to match the circumference of the largest star in the system, A, as closely as possible while ensuring the entirety of the star was in the aperture. Measurements were taken from the center of the main star, A, and extended to the center of the companion star, B or C, Figure 2. Each image was measured by two group members, resulting in two sets of data for each of the AB and AC pairs. The 28 sets of data for each pair were exported to an excel spreadsheet, averaged, and used for the remaining analysis.

The data analyzed includes the proper motion RA and DEC, parallax, differential magnitudes ( $\Delta$ -mag), separation ( $\rho$ ), and position angle ( $\Theta$ ). Separation data was used to determine the apparent separation. Parallax was used to determine the actual separation and probabilities of each pair being within 1-light year of each other. Proper motion RA and DEC were used to determine the Harshaw Statistic, which gives a probability that the pair of stars are gravitationally bound (Harshaw 2014). Finally, a proper motion diagram was created using GAIA filters in SIMBAD, figure 3.

Test Images			
Date	Filter	Exposure Time (sec)	Number of Images
3/28/2023	ip	30	2
	ip	150	2
	ip	200	2
	rp	15	2
	rp	20	2
	rp	100	2
4/5/2023	ip	14	2
	ip	20	2
	rp	8	2
	rp	11	2
4/6/2023	ip	14	3
	ip	20	2
	rp	8	4
	rp	11	2
4/11/2023	ip	14	2
	ip	20	2
	rp	8	2
	rp	11	2

*Table 1: Dates, filters, exposure times, and the number of images taken during the calibration period of observation collection*

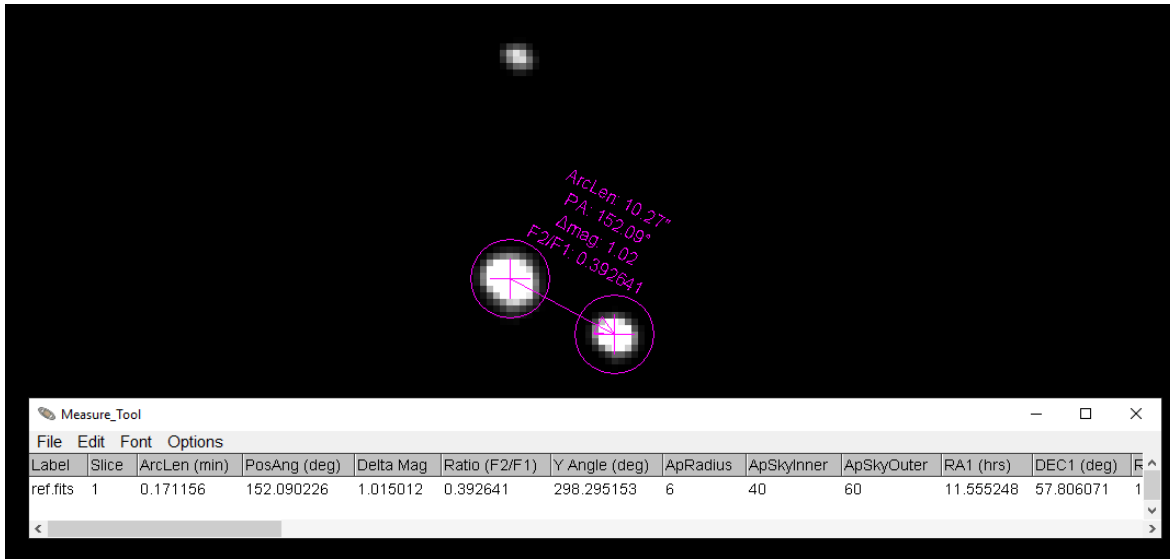


Figure 2: Screenshot of how measurements were taken in the AIJ software. Notice that the aperture fits just around the largest star in the system and each measurement is centered on the target star. A snippet of the data provided by these measurements can be seen at the bottom of the image

### 3. Results

The results in this section are the average of each team member’s independent measurements of each star pair. Table 2 outlines the dates, location, filter, and number of images taken that were the basis of the following measurements. Table 3 is the theta, rho, and delta-magnitude values of the AB pair in the MLR 551 system. Table 4 is the same type of data given for the AC star pair in the system. The mean, standard deviation, and standard deviation of the mean are given in table 4 as calculated using excel spreadsheet’s statistical analysis.

WDS Number:	11333+5748	Disciverer Code:	MLR551		
Date	Epoch	Observatory	Filter	Number	
4/23/2023	2023.309	Haleakala Observatory, Maui, USA	rp	14	
4/25/2023	2023.315	Haleakala Observatory, Maui, USA	rp	14	

Table 2: Information about the images taken and processed to extract the data used for measurement

WDS 11333+5748 MLR 551Aa,Ab KR 39AB				
Epoch	Measurement	Theta (degrees)	Rho (arcseconds)	Delta Magnitude (sloan r)
2023.315	Mean	152.1	10.3	1.02
	Standard Deviation	0.122	0.00026	0.00593
	Standard Error of the Mean	0.023	0.000049	0.0011
2017	Last measurement	152	10.4	1.05

Table 3: Mean measurements and statistical errors for the AB star pair in MLR 551, measured using AIJ. “Last measurement” is the last recorded GAIA data.

WDS 11333+5748 MLR 551Aa,Ab KR39AC				
Epoch	Measurement	Theta (degrees)	Rho (arcseconds)	Delta Magnitude (sloan r)
2023.315	Mean	268.23	19.21	2.65
	Standard Deviation	0.079	0.0004	0.0093
	Standard Error of the Mean	0.015	0.000076	0.0018
2015	Last measurement	268	19	2.51

Table 4: Mean measurements and statistical errors for the AC star pair in MLR 551, measured using AIJ. “Last measurement” is the last recorded GAIA data.

#### 4. Discussion

Apparent separation between the AB and AC pairs were calculated using the rho values in tables 3 and 4. The results are given in Table 5. Parallax and distance calculations are given in Table 6 for each of the three stars in the system. We used trigonometry to determine that the shortest possible distance between the AB star pair is 0.68 light-years, and 7213.6 for the AC pair. We calculated the probability of the pairs being within 1 lightyear, finding the probability of the AB and AC pairs to be 39% and 0%, respectively. These are important data points because according to a study by the University of Hawaii Institute of Astronomy, the greatest distance between stars in a stable binary system is one light-year (Good, 2018). The AC pair is three orders of magnitude greater than this limit. The AB pair, on the other hand, is within the limit.

Another calculation performed to analyze our data comes from Harshaw (2014) using proper motion of star pairs to determine the likelihood that the stars are gravitationally bound. Using this method of statistical analysis, the AB pair has a rating of 0.09 while the AC pair has a rating of 0.67. Smaller ratings using this method suggest a higher probability of stellar pairs having orbital solutions, meaning they are gravitationally bound binary pairs. Larger values suggest that the pairs are optical binary stars.

The last method we used to study the binary nature of the MLR 551 system was a qualitative analysis of the proper motion of each star, extracted from the Aladin Sky Atlas (CDS 2000). Figure 3 shows the proper motion diagram we used to understand the motion of the stars in a two-dimensional reference frame. Each motion vector is labeled with the star it is representing. It can be seen clearly that the Aa and B stars are moving in a nearly parallel manner while the C star is moving in a near perpendicular manner. While this is only a two-dimensional representation of the star’s motion, it gives a clear picture of the binary possibilities within the MLR 551 system.

The proper motion diagram as well as proper motion RA and DEC shows that the Ab star is not moving parallel to Aa or B stars. Considering Aa and Ab have a separation angle of 0.948 arcseconds, this is an unexpected behavior because all other data suggests the stars are a close binary pair. Furthermore, we would have expected the proper motions of Aa and B stars to have been closer in a binary pair. It is possible that the Ab star is influencing a gravitational force on the Aa star just enough to alter the current trajectory. As seen in Figure 3, Ab is to the left of Aa, and the proper motion is skewed in this same direction from B star’s proper motion. We suggest this Aa,Ab pair be further studied using more appropriate methods such as speckle interferometry to determine the orbit and how it may be influencing the Aa star as well as the system as a whole.

	<u>X (arc sec)</u>	<u>d (light years)</u>	<u>1/206265</u>	<u>Apparent Separation (ly)</u>	<u>Apparent Separation (AU)</u>
<b>AB</b>	10.3	333.0272144	4.84813E-06	0.016629968	1051.695794
<b>AC</b>	19.21	7546.296296	4.84813E-06	0.70280635	44446.1764

Table 5: The apparent separations of the AB and AC pair of stars in the MLR 551 system given in light-years (ly) and astronomical units (AU). X and d are the distances as indicated by the associated units

Results	Parallax		Parsecs			Light Years		
	msecs	Std Error (SEM)	- 1 Std Error (SEM)	Mean	+ 1 Std Error (SEM)	- 1 Std Error (SEM)	Mean	+ 1 Std Error (SEM)
Star A	9.799	0.0526	101.51	102.05	102.60	330.911	332.687	334.482
Star B	9.779	0.0123	102.13	102.26	102.39	332.949	333.367	333.787
Star C	0.432	0.0117	2253.78	2314.81	2379.25	7347.307	7546.296	7756.365

Table 6: The parallax and distance to each of the three stars in the MLR 5512 system

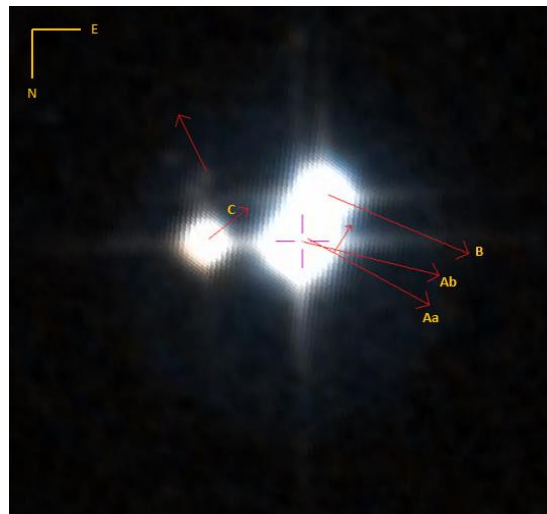


Figure 3: A proper motion diagram of MLR 551 depicting the proper motion of each of the stars in the system. Notice the odd motions of Ab compared to Aa and even B

## 5. Conclusion

The radial distance and proper motions of Aa and B stars in the MLR 551 system suggest that they are gravitationally bound. While there is currently no orbital data to study, further research on the orbital motions of these stars will likely confirm this. The C star in this system, however, cannot be determined to be gravitationally bound. The distance between it and the other stars along with the near perpendicular proper motion suggest that this star is only optically near the other stars. We therefore conclude that it is highly probable that Aa, Ab and B are a trinary star system, rich in future research opportunities and that C is an optical binary star, having no gravitational influence on the trinary system. We also propose that the system is currently mislabeled as a four-star system (Aa, Ab, B, and C), and should be relabeled as a trinary system containing Aa, Ab, and B stars.

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